The History of Electromagnetics as Hertz Would Have Known It

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Abstract —Highlights of the separate developments of the sciences of electrostatics and magnetostatics are traced through the end of the 18th century, climaxed by the work of Coulomb and Poisson. The linkage of these two sciences due to the discoveries of Oersted, Ampère, Biot and Savart, and Faraday are described, followed by the theoretical culmination embodied in the work of Maxwell. His prediction of the existence of electromagnetic waves is seen to set the stage for the epochal experiments of Hertz.

I. Introduction

As WE REACH the centennial of Hertz's validation of key predictions arising from Maxwell's theory—namely that time-varying electric currents can produce electromagnetic waves which travel at the speed of light—it seems fitting to look back on the series of discoveries involving electrical phenomena that led to the crowning achievement of Maxwell. A rich body of knowledge was available to Hertz which he understood fully. Thus he was blessed from the outset by an awareness of what he was seeking, which in no way diminishes his truly seminal accomplishment.

In preparing this historical overview, the author has drawn on many sources, principally Whittaker's *A History of the Theories of Aether and Electricity* (London: T. Nelson and Sons). He has also excerpted liberally from his own text *Electromagnetics* (New York: McGraw-Hill).

II. Electrostatics Before 1700

The ancients were familiar with a curious property of the mineral amber, namely that after being rubbed it attracted light bodies. For 2000 years this was regarded as a property peculiar to amber. By the 17th century it was recognized that this view was incorrect. William Gilbert, physician to Queen Elizabeth, noted the same effect could be produced by friction in a large class of bodies, including glass, sulfur, sealing wax, and a variety of precious stones [1]. This attractive force, exhibited by so many different kinds of matter, needed a name; Gilbert chose to call it *electric*, an appellation that has persisted ever since.

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Niccolo Cabeo (1585–1650) may have been the first to observe that electrified bodies repel as well as attract.[1] Except for a theory proposed by Gilbert, that the rubbing of an electrifiable body caused an effluvium to issue from the body, which formed an attracting or repulsing atmosphere around it, there was little theoretical musing on the subject prior to 1700. However, many commonplace occurrences of static electricity were observed and recorded. Jeans [2] recounts the experience of Robert Symmer, who was in the habit of wearing two pairs of socks, the inner worsted, the outer silk. When pulling off his stockings preparatory to retiring, he noticed that they gave off a crackling noise, at times even emitting sparks. Upon separating the worsted and silk stockings, Symmer found that each became inflated so as to reproduce the shape of the foot, and exhibited attractions and repulsions at distances as great as a foot and a half.

III. Magnetostatics Before 1700

The ancients were acquainted with a similar property possessed by the mineral called lodestone, which (without rubbing) had the power of attracting iron. This led to the invention of the compass, but where, when, and by whom is not known. The earliest reference to it is in a work by Alexander Neckam (1157–1217), a monk of St. Albans, who does not treat it as something new.[2]

The *science* of magnetism can be said to date from 1269, for in that year Pierre de Maricourt (Peregrinus) announced the discovery of an important property of lodestones. In his own words [3],

> So you must know that this stone bears in itself the similitude of the heavens, the method of proving which I will explain clearly how to find... there are two points in the heavens more noteworthy than the rest, because the celestial sphere turns

[1] This discovery was reinforced in 1733 by du Fay, who established that there were two types of electricity, which he called vitreous and resinous Benjamin Franklin, in 1747, unaware of the work of du Fay, independently made the same discovery and attached the labels plus and minus to the two types of electricity, a notation that has been universally adopted.

[2] A summary of what is known about the discovery of the compass, with bibliography, has been given by D. G. Knapp in "Origins of Geomagnetic Science," ch. 6 of *Magnetism of the Earth*, Publication 40-1, Coast and Geodetic Survey, U.S. Dept. of Commerce, Washington, DC, 1962.

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> about them as upon axes. One of these is named the Arctic or North pole, whilst the remaining one is named the Antarctic or Southern. So in this stone you should thoroughly comprehend there are two points of which one is called the North, the remaining one the South. To the general discovery of these two points you may attain by manifold industry...one way is to have this stone rounded with a tool with which crystals and other stones are rounded. Afterwards let a needle...be placed over the stone, and along the length of the needle let a line be marked out dividing the stone along the middle. Afterwards let the needle be placed in another position over the stone, and mark the stone with a line again in the same way according to that position. And if you wish, you shall do this in several places or positions, and without doubt all the lines of this kind will meet in two points, just as all the meridian circles of the World meet in the two opposite poles of the World. Know you then that one is the North, the other the South.

Peregrinus went on to the discussion of further experiments in which he showed that the two poles were also the points of greatest concentration of magnetic strength. His terminology has prevailed to this day, and this conception of a polarization effect in magnets has made a lasting impression and forms the basis for many subsequent theories of magnetization.

Gilbert was the first to appreciate that the earth itself is a giant spherical magnet. He went so far as to magnetize a small iron ball and demonstrate that it possessed a magnetic field similar to that of the earth. The action of a compass needle was then readily explained as merely another example of the principle that like poles of different magnets repel, whereas unlike poles attract [1].

The 17th century witnessed a widened interest in magnetic investigations. Among the accomplishments of that period may be mentioned the demonstration by A. Kirchner (1601–1680) that the two poles of a magnet have equal strength. This was done by measuring the force required to pull a piece of iron away from either pole. N. Cabeo revealed an inductive effect when he noted that an *unmagnetized* needle floating freely on water would align itself with the earth's magnetic meridian. H. Gellibrand (1597–1636) discovered the secular variation of the magnetic declination. Descartes offered the first theoretical explanation of magnetic phenomena by attempting to embrace all known effects within his theory of vortices. He assumed that the fluid matter of a vortex entered a magnet at one pole and emerged at the other, acting on nearby pieces of iron because the molecules of the iron presented a special resistance to its motion.

IV. Electrostatics in the 18th Century

The pace of discovery of both electric and magnetic phenomena quickened shortly after 1700. The discovery that certain materials could be used to convey electricity from one place to another was made by Stephen Gray [4] in 1729. His experiments were quasi-static and originally dealt with a glass tube about three feet long, to one end of which he fitted a cork. Upon rubbing the glass tube Gray found that the cork also became electrified, and concluded that "there was certainly an attractive Vertue communicated to the Cork by the excited Tube." Stimulated by this result, Gray interposed a wooden rod between the glass tube and cork and observed the same effect. Next he connected tube and cork with iron or brass wire. Still the same effect, undiminished by the length of wire. Finally, he tied one end of a length of hemp cord to the glass rod and the other end to an ivory ball. Using lengths of cord as great as 400 feet, Gray was able to electrify the ball by rubbing the distant glass rod.

Among those to whom Gray first communicated this discovery of electrical conduction was J. T. Desaguliers (1683–1744), who continued the experiments after Gray's death in 1736. Desaguliers determined [5] that only a limited class of materials, notably the metals, could convey electricity easily and to these materials he gave the name conductors. As a consequence those nonmagnetic materials which proved to be poor conductors became known as insulators.

Several one-fluid and two-fluid theories came into vogue in attempting to explain the nature of electricity, but they served more to confuse than enlighten. The real advances were quantitative as the result of experimentation. Notably, the inverse square law for the force between electrified particles was firmly established in the 18th century.

This law has a curious history of discovery and rediscovery. As is true with respect to most major scientific principles, its establishment cannot be wholly credited to the efforts of one man. Perhaps the first significant contribution to the realization of this law was made by Benjamin Franklin (1706–1790). Writing to Dr. John Lining of Charlestown, South Carolina, on March 18, 1755, Franklin described an experiment he had performed in the following words [6]:

> I electrified a silver pint cann, on an electric stand, and then lowered into it a cork-ball, of about an inch diameter, hanging by a silk string, till the cork touched the bottom of the cann. The cork was not attracted to the inside of the cann as it would have been to the outside, and though it touched the bottom, yet, when drawn out, it was not found to be electrified by that touch, as it would have been touching the outside. The fact is singular. You require the reason; I do not know it. Perhaps you may discover it, and then you will be so good as to communicate it to me.

Later, upon editing a collection of his letters for publication, Franklin added the footnote

> Mr. F. has since thought, that, possibly the mutual repulsion of the inner opposite sides of the electrised cann, may prevent the accumulating an electric atmosphere upon them and occasion it to stand chiefly on the outside. But recommends it to the farther examination of the curious.

Very little progress was made with this idea until Franklin described the above-mentioned experiment to his good friend Joseph Priestley and asked Priestley to repeat the investigation and verify his results. Priestley (1733–1804), better known as the discoverer of oxygen, undertook experiments beginning in December 1766. He suspended two pith balls from threads which were entirely inside an electrically charged cup. Like Franklin, Priestley found [7] that the balls

> remained just where they were placed, without being in the least affected by the electricity; but that, if a finger, or any conducting substance communicating with the earth, touched them, or was even presented towards them, near the mouth of the cup, they immediately separated, being attracted to the sides; as they also were in raising them up, the moment that the threads appeared above the mouth of the cup.

Based on the results of the experiment, Priestley then made the following observation:

> May we not infer from this experiment, that the attraction of electricity is subject to the same laws with that of gravitation, and is therefore according to the square of the distances; since it is easily demonstrated that were the earth in the form of a shell, a body in the inside of it would not be attracted to one side more than another.

Despite the fact that Priestley was prompt to publish these experimental findings and his inference of the inverse square relation, the scientific community of his day failed to appreciate the significance. Indeed, Priestley himself apparently did not regard this accomplishment as a sufficiently rigorous proof and did not champion his deductions.

Two years later, in 1769, Dr. John Robison (1739–1805), of Edinburgh, undertook the task of determining the law of force between electric charges by direct experiment. Little attention has been given to the historical priority of his discovery, since Robison made scant attempt at the time to publicize his findings. This is unfortunate, because he was an accomplished investigator of wide interests, whose discoveries could have benefited the progress of science. His lectures and scientific researches were published posthumously in Edinburgh in 1822 and are clearly and engagingly presented in an extensive four-volume treatise entitled *Mechanical Philosophy*. Commencing on page 73 of the fourth volume of this treatise, Robison describes in detail an electrometer which he constructed for the purpose of detemining the force law between electrified particles. Fig. 1 is a reproduction of Robison's sketch of the electrometer, a device which balances gravitational and electrical forces. Noting that he had made many hundreds of measurements with different instruments, Robison concluded that

> the mutual repulsion of two spheres, electrified positively or negatively, was very nearly in the inverse proportion of the squares of the distance of their centres, or rather in a proportion somewhat greater, approaching to $1/r^{2.06}$.

By rotating the apparatus so that B was *under* A, Robison was able to make measurements of the attractive force between unlike charges. The results were similar and he concluded that the force law was probably the inverse square of distance for both attraction and repulsion. He failed to recognize the importance of this result, perhaps because of the subordinate position in which he tended to place experimental work relative to mathematics.

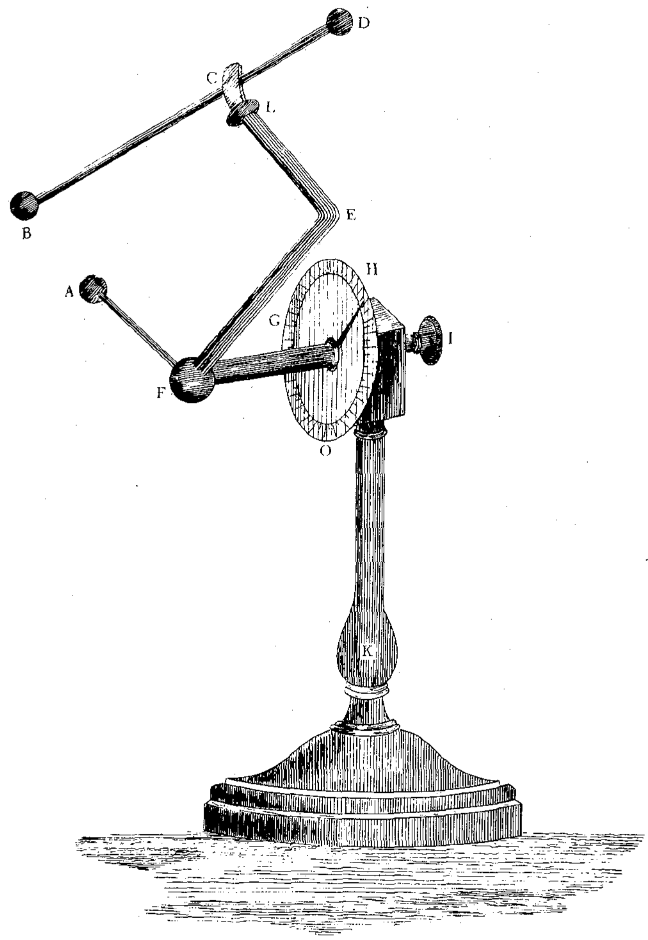


Fig. 1. Robison's apparatus.

Another definitive demonstration of the inverse square law was achieved by Henry Cavendish (1731–1810) in 1773. His experiment had the same basic form as the approach used earlier by Franklin and Priestley, although it is not clear that Cavendish was aware of their efforts. He went far beyond their accomplishments, however, and obtained a quantitative result for the law of force, including an estimate of the precision of his data.

The laboratory technique displayed by Cavendish in all his researches would earn the admiration of any modern experimenter. In his earlier work with electricity, he had developed the concept of "degree of electrification" (now called potential), and had then convinced himself that when two charged conductors are connected by a wire they redistribute charge in order to attain the same potential. He incorporated this result into many experiments designed to compare the charge on two bodies which had been brought to a common potential.

In one of these experiments, Cavendish showed that the charges on similar bodies at the same potential are in the ratio of their linear dimensions. Using this knowledge, he expressed the charge on any body in terms of the diameter of a sphere which, when at the same potential, would have an equal charge. This, in modern language, is the concept

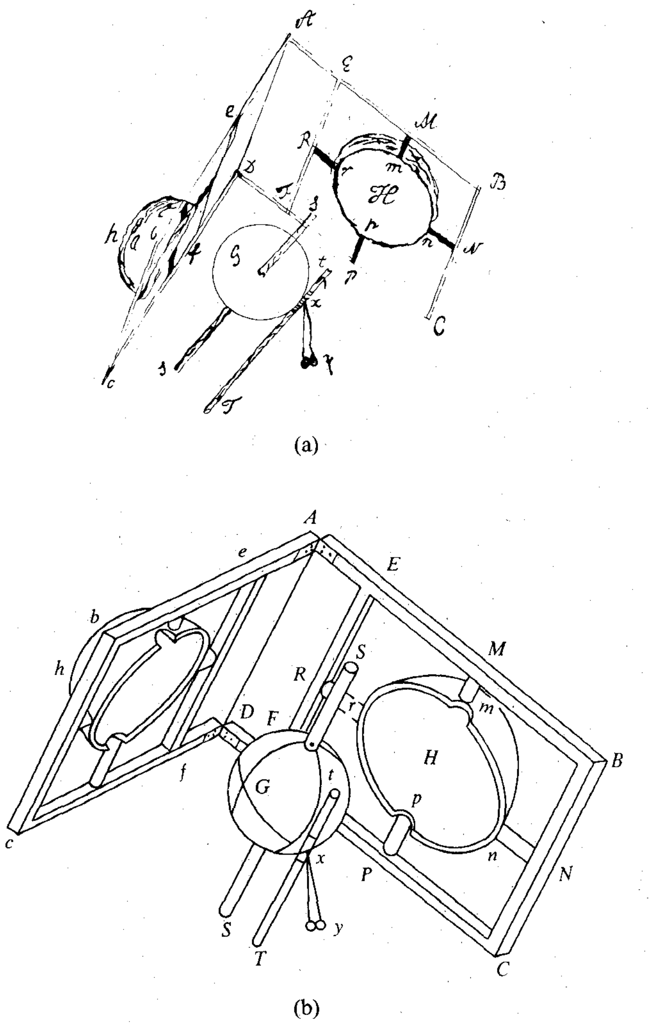


Fig. 2. The Cavendish apparatus. (a) Cavendish's original sketch. (b) Maxwell's drawing.

of capacitance, and when Cavendish spoke of the charge of a body as "globular inches" or simply "inches of electricity" he meant that the capacitance of the body in question was equal to that of a sphere whose diameter in inches was the value quoted. Cavendish took as his standard a conducting spherical shell whose diameter was 12.1 in. and he then ascertained, by a well-arranged series of measurements, the relative capacitances of a great number of bodies of many shapes.

His electric force experiment had the intention [8]

> to find out whether, when a hollow globe is electrified, a smaller globe inclosed within it and communicating with the outer one by some conducting substance is rendered at all over or undercharged; and thereby to discover the law of the electric attraction and repulsion.

To this end, Cavendish constructed an apparatus consisting of a 12.1 in. diameter inner globe, mounted on a glass rod, and surrounded by two hemispheres of diameter 13.3 in., the latter mounted in a hinged wooden frame, as shown in Fig. 2. He then

> made a communication between them by a piece of wire run through one of the hemispheres and touching the inner globe, a piece of silk string being fastened to the end of the wire, by which I could draw it out at pleasure.

Cavendish next charged the outer globe, withdrew the connecting wire, removed the two hemispheres, and tested for charge on the inner globe by touching to it an electometer consisting of two pith balls suspended by fine linen threads. He observed that

> The result was, that though the experiment was repeated several times, I could never perceive the pith balls to separate or shew any signs of electricity.

Cavendish went on to demonstrate that the amount of electricity transferred from the outer globe to the inner was less than $1/60$ of the total and deduced that

> the electric attraction and repulsion must be inversely as some power of the distance between that of the $2+1/50$th and that of the $2-1/50$th, and there is no reason to think that it differs at all from the inverse duplicate ratio.

Since Cavendish had shown that the charges on similar bodies at the same potential are in the ratios of their linear dimensions, it was a simple matter for him to halve the amount of charge on two identical electrometers. He subsequently found the force to be reduced by a factor of four and concluded that the electric force was linearly proportional to the amount of charge.

Cavendish's approach to the demonstration of the electric force law is important because of its inherent accuracy. It was repeated by Maxwell a century later [8], with the result that the dependence on separation distance was bracketed by $r^{-2\pm\delta}$, with $\delta \leqslant 1/21600$. In 1936, Plimpton and Lawton [9], using state-of-the-art measurement techniques, showed that $\delta \leqslant 2\times 10^{-9}$.

The results of Cavendish's highly original and definitive experiments were unknown to the scientific community for almost a century, for, like Robison, Cavendish chose not to publicize his findings. By the time a general awareness had developed that each of these men had established the inverse square law, the credit and fame had been bestowed properly on someone else.

That someone else was Charles Augustin de Coulomb (1736–1806), who, in 1785, also demonstrated the law of electric force, using a technique totally different from those employed by any of his predecessors. Coulomb's procedure involved the use of a torsion balance which he had invented. With it, he measured the repulsive force between two like charges, balancing this force by the torsion in a wire from which a bar containing one of the charges was suspended.

Coulomb's drawing of the original apparatus is shown in Fig. 3. Upon charging the identical pith balls equally and alike, Coulomb could control their separation by the amount of torsion in the suspending wire. Knowing the torque/torsion relation, he was able to deduce the force law. Coulomb published *three* data points (!), one of which deviated from the inverse square law by 6 percent [10]. But he undoubtedly had performed many more trials. He was

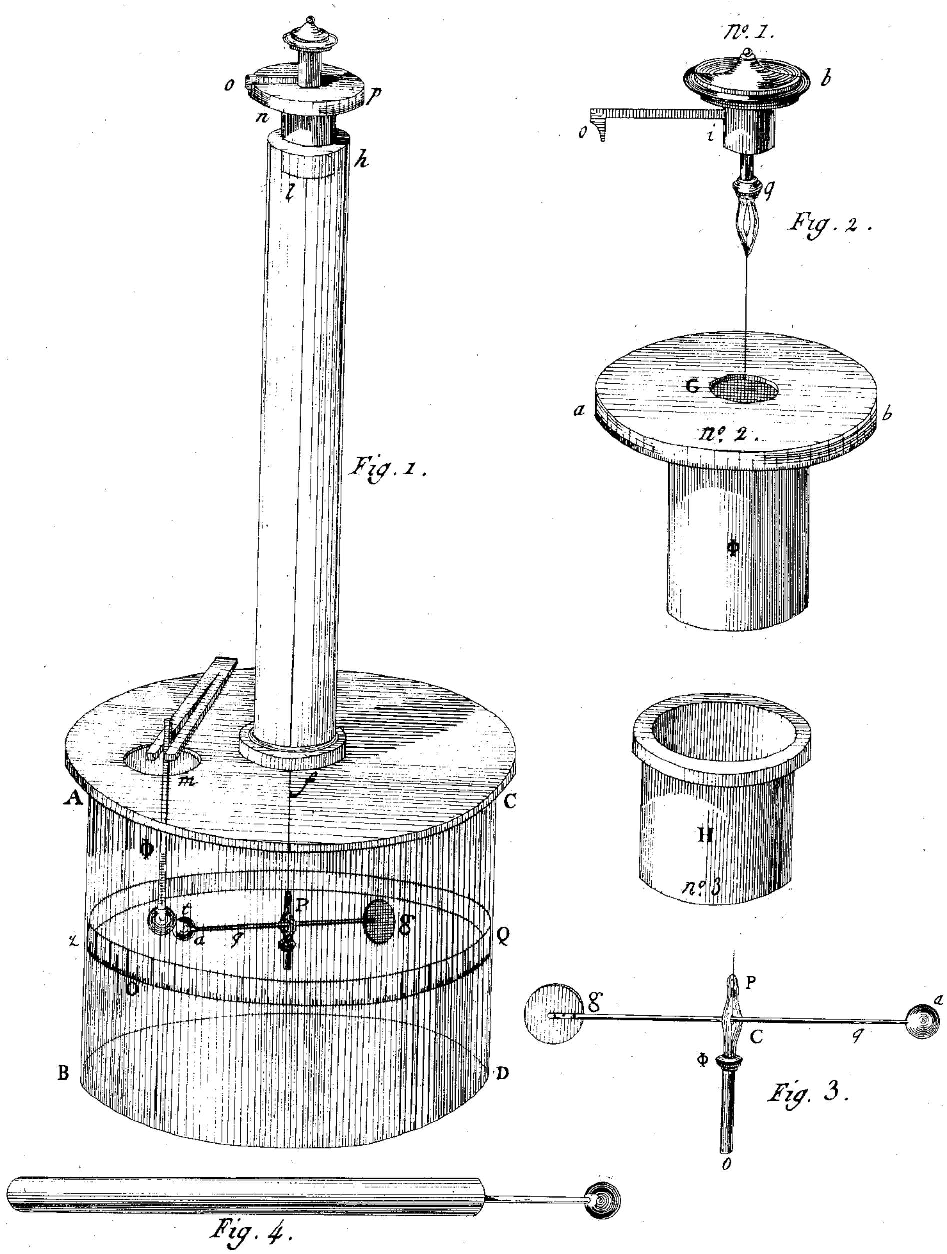


Fig. 3. Coulomb's apparatus.

led to the conclusion

> It results then from these three trials that the repulsive action which the two balls exert on each other when they are electrified similarly is in the inverse ratio of the square of the distance.

Coulomb devised an equally novel apparatus [11] to determine the force law for unlike charges. It consisted of an oscillating horizontal rod suspended from a silk thread, as shown in Fig. 4, placed proximate to a large charged sphere. With the disc l oppositely charged to the sphere, a restoring electric force arises when the rod is set into oscillation, the period of which is related to the restoring force. Coulomb was able to deduce from experiments using this apparatus that once again the force law involved the inverse square of the distance of separation of the charges. By a technique equivalent to the one used by Cavendish, he was also able to show that when either charge was halved, the force was also halved.

Coulomb's publications received prompt and broad circulation and his findings were accorded widespread acceptance. The force law for electrified particles was now on an equal footing with Newton's gravitational law for mass. The quantitative age for electrostatics had begun with the equation

$$f \propto qq'/r^2 \tag{1}$$

thereafter known as Coulomb's law.

V. Magnetostatics in the 18th Century

The taxonomy of materials, which at first contained the two general classes, magnetic and nonmagnetic, began to assume more detail in the 1700's. We have seen that Gray's discovery of electrical conduction led to the division of

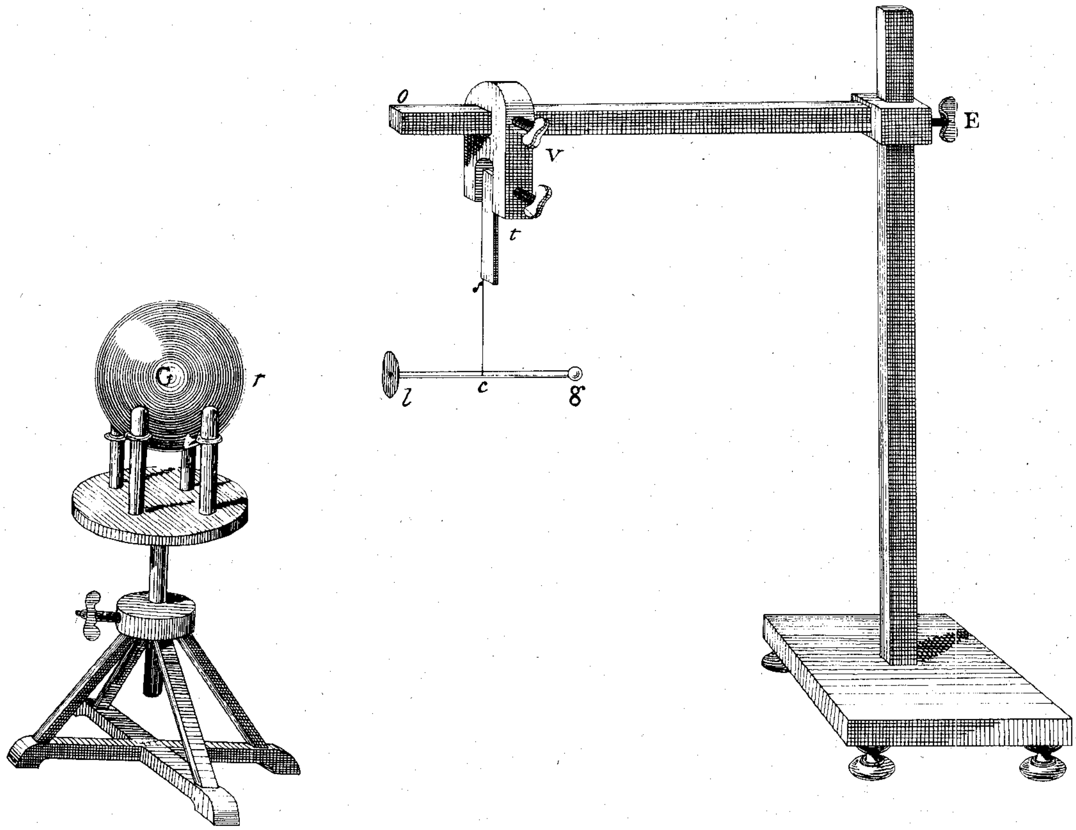


Fig. 4. Coulomb's apparatus for unlike charges.

nonmagnetic materials into two types, conductors and insulators, with many examples of each having been identified. Parallel to this, lodestone had been joined by a growing list of iron ores which were known to possess magnetic properties. "Artificial" magnets had also been produced by various investigators.

It was widely appreciated that many distinctions existed between electric and magnetic materials. A lodestone required no frictional stimulus such as was needed to stir an insulator into electrostatic activity. The lodestone attracted only magnetizable substances, whereas electrified bodies attracted virtually everything. The magnetic attraction between two bodies was unaffected by the interposition of paper or cloth, or by immersing the bodies in water, whereas electric attraction was nullified by interposing a metallic screen. Finally, a magnetic force tended to arrange bodies in definite orientations, an effect not duplicated by an electric force.

Magnetostatics was also entering the quantitative stage. Newton [12] had speculated that the law of force for a bar magnet was the inverse cube of the distance of separation.[3] However, John Michell (1724–1793) was the first to enunciate a correct law for the force between magnetic poles, stating [13]

> Whenever any Magnetism is found, whether in the Magnet itself, or any piece of Iron, etc., excited by the Magnet, there are always found two Poles, which are generally called North and South.... Each Pole attracts or repels exactly equally, at equal distances, in every direction.... The Attraction and Repulsion of Magnets decreases, as the Squares of the distances from the respective poles increase.

Michell based the statement of this law on his own experimental observations and those of several contemporaries. The validity of the inverse square relationship was later reinforced by the refined experiments of Coulomb using the same torsion balance with which he had established the law for electric force [11].

The prevalence at that time of fluid theories of electricity naturally led to efforts to construct similar theories of magnetism. A one-fluid theory was proposed by Aepinus in 1759, in which the poles were presumed to be regions in which the magnetic fluid was present in excess or deficiency of the normal amount. A two-fluid theory was favored by Brugmans and Wilcke, with elements of one fluid repulsing each other, but attracting all elements of the other fluid. The names *austral* and *boreal* were given to these fluids and Coulomb adopted this two-fluid idea, using it to explain why a magnet, upon being broken in two, becomes two magnets, each with a pair of poles, rather than two half-magnets, each with a single pole. According to Coulomb, [14] this effect could be explained by imagining the two magnetic fluids to be trapped in equal amounts within the molecules of magnetic bodies, with no possibility of transfer of either fluid from one molecule to the next. In the unmagnetized state every

[3] This is correct for distances large compared with the length of the bar magnet.

molecule of the body has both its fluids uniformly distributed, and magnetization occurs when the austral and boreal fluids retreat to opposite ends of each molecule. This proved to be an influential idea and became known as the polarization hypothesis.

VI. The 19th Century

By 1800 the experimental method was widespread, measuring instruments were considerably improved, and new discoveries began to follow quickly on the heels of their predecessors. With the inverse square law of electric force established, a large body of mathematics already applicable to gravitational phenomena was quickly transferred and extended. The 19th century was to eclipse all earlier time in presenting advances in the electrical sciences.

Fittingly, the year 1800 was the setting for the newest discovery, since it was in that year Volta announced his invention of the first chemical battery [15]. Motivated by Galvani's researches on animal electricity, Volta developed a pile consisting of pairs of strips of dissimilar metals immersed in brine or a weak acid electrolyte. When a circuit was formed by connecting a wire across the pairs of strips, a *continuous* electric current was observed to flow. This was one of the most important discoveries in the history of electrical science, and led immediately to a profusion of fruitful investigations. Indeed the announcement of Volta's invention was so startling that scientists on both sides of the Atlantic set forth to repeat and extend his experiments. In England, Nicholson and Carlisle constructed a voltaic pile and with it effected the electrical decomposition of water into its constituent gases. This achievement was then extended by Cruickshank, who demonstrated that metallic salt solutions could be similarly decomposed. Wollaston next showed that water could also be decomposed by a discharge of frictional electricity, thus inferring that the sources of voltaic electricity were common with those of electrostatic phenomena.

These experiments attracted the attention of Humphry Davy (1778–1829), who at about this time was appointed Professor of Chemistry at the Royal Institution in London. Together with William Pepys, an instrument maker and Fellow of the Royal Society, Davy designed and had constructed a succession of voltaic piles which were the largest then in existence. The last of these was built in 1808 and consisted of 2000 pairs of plates of zinc and copper, each plate being 6 in. square. With these batteries, Davy melted iron wires up to a tenth inch in diameter and decomposed alkalis, obtaining thereby potash and soda ash from which he extracted the new elements potassium and sodium. He was also to melt quartz, sapphire, and platinum, to evaporate diamond, and to boil liquids such as water and oil. The new elements barium, strontium, magnesium, and calcium were extracted from the decomposition of alkaline earths. And Pepys in 1815 utilized the intense heat developed by the voltaic pile to melt iron wire and diamond dust together, thus directly carburizing the iron and producing steel.

In 1821 Davy turned his attention to the problem of determining the ability of various metals to conduct a voltaic current [16]. He accomplished this by connecting a voltaic battery across a circuit consisting of a column of water in parallel with the metallic wire being investigated. When the length of wire was less than a certain critical value, the division of current was such that the water ceased to decompose. Davy measured the lengths and weights of wires of different materials which would cause this critical condition; by comparing the results he was able to show that the critical conductance of a wire was inversely proportional to its length l and directly proportional to its cross-sectional area A, though independent of the shape of the cross section. Critical conductance could thus be expressed by the formula $G = \sigma(A/l)$, in which σ is a fundamental material property called the electrical conductivity. With this apparatus Davy also was able to compare the conductivities of different metals, and determined additionally that critical conductivity varied inversely with temperature.

A year earlier Ampère had provided a usable definition for current and devised an instrument for measuring it, which he called a galvanometer. He distinguished between electric tension (voltage) and electric current, and observed that electric tension existed in a voltaic battery before the circuit was closed, being detectable through the use of an electroscope. He viewed tension as a cause and current as an effect. Ampère realized that a relation existed between the cause and the effect, but neither he nor Davy appreciated that the relation was a simple ratio in proportion to Davy's critical conductivity figures. This final link in the chain would be provided by Ohm six years later.

Meanwhile electrostatic and magnetostatic theory were being firmly based and brilliantly advanced by Siméon Denis Poisson (1781–1840). He did this in two memoirs [17] which used Coulomb's inverse square law as a fundamental postulate and made rich use of the analogy to gravitational theory, a subject already highly advanced at that time.

In an article in the *Mémoires de Berlin* in 1777, Lagrange had shown that if a function $\psi(x, y, z)$ were formed by adding together the masses of all the particles of an attracting system, each divided by its distance from (x, y, z), then the derivatives of this function were equal to the components of the attractive force at (x, y, z). Laplace later demonstrated [18] that this function ψ satisfies the equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = 0$$

at all points not occupied by masses.

In like manner, Poisson introduced a function[4] $\Phi(x, y, z)$, composed of the sum of the charges of an electrical system, each divided by its distance from (x, y, z). He then

[4] Fifteen years later, in generalizing Poisson's work on electric and magnetic phenomena, George Green (1793–1841) gave to this function the name *potential*.

argued, as had Lagrange in the case of gravitational attraction, that the derivatives

$$-\frac{\partial \Phi}{\partial x} \qquad -\frac{\partial \Phi}{\partial y} \qquad -\frac{\partial \Phi}{\partial z}$$

would yield the components of electric force[5] at (x, y, z).

Turning his attention to conducting bodies, Poisson assumed that an excess of one type of charge had been placed on a conductor and reasoned that, in equilibrium, it would have to be distributed on the surface, and in such a way that no electric force existed at any interior point of the conductor. He then claimed that

> the value of Φ is independent of the coordinates of the point P; because then the partial derivatives of this function being null, the force at the interior point P will be also.

Thus was the concept formulated that a conducting body in electrostatic equilibrium is an equipotential.

Poisson next turned his attention to conditions at the surface of an electrified conductor and argued, following a suggestion by Laplace, that the electric force at a point immediately outside the conductor is proportional to the local concentration of surface charge density. He did this by dividing the force into a part f due to the element of charged surface immediately adjacent to the point, and a part F due to the rest of the surface. At a neighboring point just *inside* the conductor, F will be unchanged but f will have to be reversed to give a null force. Therefore the resultant force at the exterior point must be $2f$. But if the exterior point is extremely close to the surface, the immediately adjacent surface element looks like an infinite plane, uniformly charged, for which case Poisson showed the force f to be proportional to the charge per unit area of the surface.

Using the principle that a charged conductor must be an equipotential, Poisson deduced the surface distribution for several simple shapes, including an ellipsoid, and then enlarged his analysis to the study of two charged spheres placed at any distance from each other. This was a classic and difficult problem to which he devoted over three quarters of the space occupied by two lengthy memoirs. The solution involves single or double gamma functions, depending on whether or not the two spheres are in contact. Poisson laboriously computed the values of his integrals for a variety of conditions and exhibited very satisfactory agreement with the earlier experimental results of Coulomb.

The year 1813 recorded another significant contribution by Poisson when, in a brief note [19], he extended Laplace's equation to include points occupied by matter, obtaining

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = -4\pi\rho \qquad (2)$$

in which ρ is the volume density of mass. The same connection exists, of course, between electric potential and charge density. Poisson's proof of the validity of this important differential equation, which bears his name, has a simple elegance which will fully reward a decision to consult the original paper.

Poisson's differential equation, linking spatial derivatives of the electrostatic potential to charge distribution, found its integral counterpart through a discovery by Karl Friedrich Gauss (1777–1855). In 1813 Gauss established [20] the famous divergence theorem:

$$\int_S \boldsymbol{D}\cdot d\boldsymbol{S} = \int_V \nabla\cdot\boldsymbol{D}\,dv \qquad (3)$$

connecting a volume integral throughout V to a surface integral over S, with S being the closed surface bounding the volume V, and $\boldsymbol{D}$ being any vector function possessing continuous first derivatives in a region containing V. If $\boldsymbol{D}$ is a radial field which varies inversely with distance from some point O, that is, if $\boldsymbol{D} = \mathbf{1}_r/r^2$, then the surface integral of Gauss's divergence theorem yields the simple result

$$\int_S \boldsymbol{D}\cdot d\boldsymbol{S} = 4\pi \qquad (4)$$

if O is inside V; otherwise the result is zero. This special result is known as Gauss's integral. When $\boldsymbol{D}$ is properly related to Coulomb's inverse square law, $\int_S \boldsymbol{D}\cdot d\boldsymbol{S}$ equals the net charge enclosed by S. This result, coupled with the divergence theorem, yields an integral form of Poisson's equation.

Poisson also made a significant contribution to the theory of behavior of magnetic materials [21]. He adopted Coulomb's model of two magnetic fluids trapped in a molecule, but moving to opposite ends of the molecule when the magnetic material became excited. This polarization of the two magnetic fluids then caused a magnetic field distribution which was derivable as the gradient of a potential function Φ_m. Poisson showed this potential function to be given by the expression

$$\Phi_m = \int_S \frac{\boldsymbol{M}\cdot d\boldsymbol{S}}{r} + \int_V \frac{(-\nabla\cdot\boldsymbol{M})}{r}\,dV \qquad (5)$$

with the first integral taken over the surface of the magnetic body, the second taken throughout its volume, and $\boldsymbol{M}$ the polarization density, or magnetization. This formula shows that the magnetic field produced by the body is the same as would be caused by a fictitious distribution of magnetic charges, consisting of a surface layer whose density is the normal component of $\boldsymbol{M}$, plus a volume distribution of density $-\nabla\cdot\boldsymbol{M}$. With this interpretation, Poisson was able to explain the magnetic phenomena known as that time. Equation (5) would find more proper application to the theory of the behavior of dielectric materials later in the century via the adaptations by Lord Kelvin [22] and F. O. Mossotti [23].

To this point, electrostatics and magnetostatics were entirely separate disciplines, with no apparent linkages between the two sets of phenomena. This viewpoint was abruptly shattered in the winter of 1819–1820. During that

[5]Poisson's original notation has been altered to be consistent with modern usage.

period, Professor Hans Christian Oersted (1777–1851), of the University of Copenhagen, experimented with the placement of a closed electric circuit near a compass needle. He had been motivated in this study by the observation that a compass needle fluctuated erratically during a thunderstom. Accordingly, he set up an apparatus consisting of a galvanic battery and a short-circuiting wire. Apparently during one of his lectures Oersted placed the wire at right angles to a compass needle, but observed no effect. At the end of this lecture the thought occurred to him to place the wire parallel to the needle. This action immediately caused a pronounced deflection in the needle. After putting together a more powerful galvanic battery, Oersted assembled some of his colleagues as witnesses and repeated the experiment. Noting that a rotation of the wire would be tracked by a rotation of the magnetic needle, and that no effect was observed for needles made of brass, glass, or gum lac, Oersted [24] offered a few observations in the nature of an explanation of the phenomenon:

> The electric conflict acts only on the magnetic particles of matter. All non-magnetic bodies appear penetrable by the electric conflict, while magnetic bodies, or rather their magnetic particles, resist the passage of this conflict. Hence they can be moved by the impetus of the contending powers.
>
> It is sufficiently evident from the preceding facts that the electric conflict is not confined to the conductor, but dispersed pretty widely in the circumjacent space.
>
> From the preceding facts we may likewise collect that this conflict performs circles; for without this condition, it seems impossible that the one part of the uniting conductor, when placed below the magnetic pole, should drive it towards the east, and when placed above it towards the west.

Oersted's discovery was promptly enlarged by others. The academician Arago learned of it while traveling abroad and, upon his return to Paris, described the effect at a meeting of the French Academy on September 11, 1820. This news excited the interest of several investigators, and the next discovery was announced by André-Marie Ampère (1775–1836) just one week later. Reasoning that if magnets exert forces on each other and if electric currents exert forces on magnets, then two electric currents should interact, Ampère devised an experiment [25] in which

> in parallel directions, two straight parts of two conducting wires joined the terminals of two voltaic piles; the one being fixed, and the other suspended from points and made very mobile by a counterpoise, being able to approach or withdraw while still retaining its parallelism with the first wire. I have then observed that upon passing an electric current through each of them, they mutually attract if the two currents are in the same direction, and that they repel each other when, instead, (the currents) are in opposite directions.

Meanwhile, Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841) repeated Oersted's experiments, and announced to the Academy at the October 30th meeting that they had determined a law of force which governed the effect.

The best source for the details of the experiment which established this law is Biot's *Précis Elémentaire de Physique* [26]. The method used can be understood with reference to Fig. 5, which is a reproduction of Biot's original drawing. Shown is a compass needle AB, which can freely pivot about its center point, and which is placed a distance r from a long, straight wire CMZ. A permanent magnet $A'B'$ (not shown) is positioned nearby in such a way as to cancel the effect of the earth's magnetic field. The equilibrium position of the needle is then found to be perpendicular to the wire axis. If the needle is pictured as having two equal and opposite magnetic poles at its extremities, the forces exerted by the current on these poles are thus equal, opposite, and circumferential. If then the needle is displaced from equilibrium by a small angle θ as shown in Fig. 5(b), a restoring couple is experienced by the needle, and its equation of motion is

$$-F(r)L\sin\theta = I\ddot{\theta}$$

in which L is the length of the needle and I is its moment of inertia. For small displacements, harmonic oscillations will occur of period

$$\tau = 2\pi\sqrt{\frac{I}{LF(r)}}.$$

Thus, in Biot's words,

> if we compare in this way, the squares of the periods, for different distances of the uniting wire from the needle, supposing always the condition of isochronism to be fulfilled, we shall obtain the ratios of the component forces exerted in these different cases by the uniting wire, parallel to the direction of equilibrium about which the needle oscillates.

Upon performing this experiment, Biot and Savart found that observed and calculated periods agreed quite well if $F(r)$ were assumed proportional to $1/r$.

Biot extended this experiment significantly by inquiring what the action must be on the compass needle due to an infinitesimal length of the wire. Since the influence of the entire straight wire varied as r^{-1}, and since r^{-1} is the integral of r^{-2}, he felt that each element of the wire should make a contribution to the total force which is proportional to the inverse square of its distance from the needle. However, he realized that this contribution might also depend on the orientation of the element relative to the needle, and devised an experiment to deduce this relation. With reference to Fig. 5(c), Biot introduced an additional V-shaped wire with its apex close to the central point of the first wire. He then determined the period of the compass needle as a function of r with a steady current alternately passing through the straight wire and the bent wire. The difference in period under the action of the two wires could be explained [27] by the assumption that the contribution from a single current element Idl was proportional to $(\sin\omega)/r^2$. The discovery of this fact led Biot to proclaim

> the elementary action of any lamina whatever (is) proportional in $\sin\omega/r^2$; and uniting with this expression, which is founded

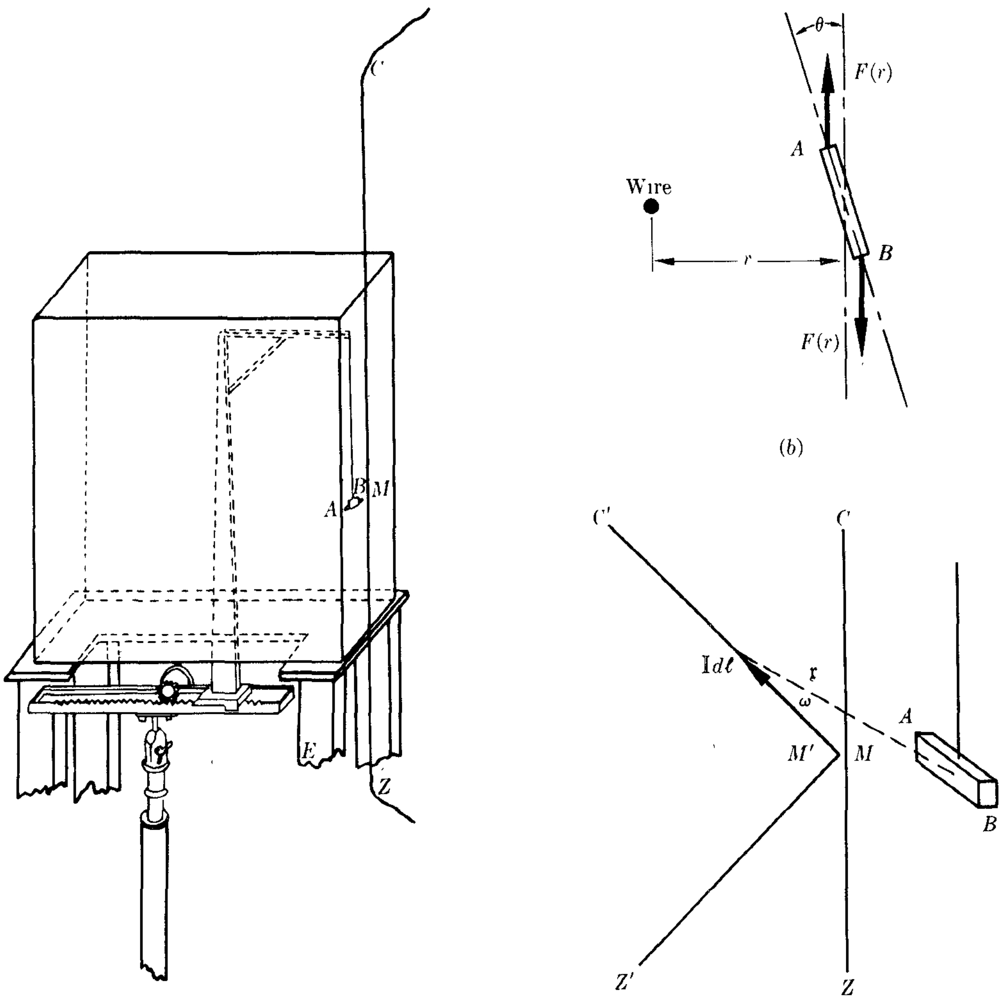


Fig. 5. The Biot–Savart experiments.

> upon experiment, the knowledge of the absolute direction of the force which is perpendicular to the plane drawn through each distance and through the direction of each longitudinal element of the wire under consideration, we may assign by calculation the total resultant of the action exerted by a wire, or by any portion of a wire, whether straight or curved, limited or indefinite.

In present-day notation, this result is equivalent to saying that a system of steady currents I creates a magnetic field at point (x, y, z) given by

$$\boldsymbol{B}(x, y, z) \propto \int \frac{I\,d\boldsymbol{l} \times \boldsymbol{r}}{r^3} \tag{6}$$

and that if a magnetic pole of strength m is placed at (x, y, z) it will experience a force $m\boldsymbol{B}$. In the above formula, r is the distance from the element dl to (x, y, z). This important equation is known as the Biot–Savart law and is often taken as the experimental postulate on which magnetostatics is based.

Ampère, following his announcement of the discovery of the force between two currents, continued his investigations and succeeded in clarifying much of what was known about electricity at that time. He distinguished phenomena involving electricity at rest from phenomena involving electricity in motion, introducing for the former the name electrostatics, and for the latter the name electrodynamics. He also distinguished between electric tension (voltage) and electric current. At that time, people were accustomed to speak of the conduction and flow of electricity, but since the two-fluid theory was popular, considerable confusion existed with respect to the nature of the flow process. Ampère decided that he would call the whole process an electric current, without regard to its inner nature, and with the direction of the current defined as the direction in which the positive fluid was presumed to move. This made the electric current something definite in terms of which phenomena could be described.

The concept of electric potential, or tension, had been privately appreciated by Cavendish, and had been admirably developed for electrostatics by Poisson. Ampère noted that electric tension was observable in a voltaic pile before the circuit was closed, being detectable through use of an electrometer of electroscope, instruments which Ampère labeled as measures of tension. As for the current itself, Apmère felt that it was best measured by means of its magnetic effects, and he introduced for this purpose an instrument which he called a galvanometer, an instrument which is still in use today.

To Ampère, tension appeared as a cause, and current as an effect. Noting that as soon as the effect appears through completion of the circuit, the tension "disappears, or at least becomes very small," Ampère [25] then made the interesting observation

> The currents of which I speak self-accelerate until the inertia of the electric fluids and the resistance that they encounter due to the imperfections in even the best conductors cause equilibrium with the electromotive force, after which they continue indefinitely at a constant speed such that this force remains at a constant intensity; but they cease entirely at the instant that the circuit is interrupted.

Ohm's law, which was to be enunciated six years later, is thus seen to be not far off in Ampère's thinking.

With a clear definition of current, and a means for measuring it, Ampère continued his researches over the next three years, and in 1825 collected his results in a lengthy memoir [28] which must rank as one of the most distinguished in the history of science. In this memoir, Ampère concerned himself with the problem of determining the law of force between two current elements. A wide variety of experiments on an assortment of wire geometries had led him to four conclusions about the force interaction between currents:

1) The action of one current on another is unchanged in magnitude, but reversed in direction, when the direction of the current is reversed.
2) The effect of a conductor bent or twisted in any small manner is the same as if the contour were smoothed out.
3) The force exerted by a closed circuit on a current element is always normal to the element.
4) If all dimensions of a circuit are changed proportionally, with the currents unchanged, the forces retain their original values.

If Ampère's third condition is formulated in terms of a field concept, one may write

$$d\boldsymbol{F} = I'\,d\boldsymbol{l}' \times \boldsymbol{B} \tag{7}$$

in which $d\boldsymbol{F}$ is the force exerted on the current element $I'\,d\boldsymbol{l}'$, and $\boldsymbol{B}$ is the field caused by the closed circuit which contains $I'\,d\boldsymbol{l}$ as one of its current elements.

Ampère quite logically assumed that the force acted along the line connecting the two current elements and obtained a formula for $\boldsymbol{B}$ at variance with Biot and Savart. Once the motion of free charges under the influence of current-produced magnetic fields could be studied, the decision was clearly in favor of the Biot–Savart formula.

Unlike Biot, who regarded magnetic poles as fundamental, Ampère considered magnetism to be basically an electrical phenomenon. He viewed a magnetized rod as equivalent to a coil carrying an uninterrupted current. He showed that two solenoïds deflect each other in exactly the same way as do two magnetized rods, and was even able to show that a single current loop, when free to move, sets itself like a compass needle with respect to the earth's magnetic field. Ultimately, Ampère came to the view that every magnetic molecule is really a small permanent circular current. This viewpoint was much too advanced for his contemporaries. The meager knowledge of atomic structure would not permit the conception of permanent currents within materials without a source of power. However, the impression produced by this memoir was deep and lasting, and today Ampère's views of these phenomena form the core of magnetic theory. He is properly credited with authorship of the force law $d\boldsymbol{F} = I'\,d\boldsymbol{l}' \times \boldsymbol{B}$, even though Biot and Savart deserve citation for the correct formulation of $\boldsymbol{B}$ in terms of the current elements in a closed circuit. Ampère himself extended the applicability of this formula by showing that a permanent magnet will exert a force on a current. His achievements were truly remarkable and Maxwell, writing half a century later, labeled his memoir "one of the most brilliant achievements in science." As a fitting tribute, the unit for electric current and the circuital law linking magnetic field and current are named in his honor.

During this same period Faraday made a discovery of the greatest practical importance. His interest in electromagnetism had been aroused in April 1821 when Wollaston, a colleague at the Royal Institution, attempted to make a current-carrying wire revolve around its own axis in the presence of a magnet. Although the experiment was unsuccessful, it piqued Faraday's interest. He began by reading what had been done by Oersted, Ampère, Biot and Savart, and others, and repeated many of their experiments. Finally, upon repeating Wollaston's experiment, he noted: [29]

> Magnets of different power brought perpendicularly to this wire did not make it revolve as Dr. Wollaston expected, but thrust it from side to side.... The effort of the wire is always to pass off at a right angle from the pole, indeed to go in a circle round it; should make the wire continually turn round. Arranged a magnet needle in a glass tube with mercury about it and by a cork, water, etc., supported a connecting wire so that the upper end should go into the silver cup and its mercury and the lower move in the channel of mercury round the pole of the needle.... In this way got the revolution of the wire round the pole of the magnet.... Very Satisfactory, but make more sensible apparatus.

This was the first electric motor. The next day Faraday improved on it and shortly thereafter invented the commutator. But he left to others the reduction to practice.

We come now to the year 1826 and the work of George Simon Ohm (1787–1854). Working with deficient apparatus, Ohm was nevertheless able to perform a series of carefully devised and definitive experiments which firmly established the law of conduction which now bears his name [30]. Preliminary investigations using voltaic batteries proved unsatisfactory, because the electric tension of such cells fluctuated with time due to chemical changes. For this reason Ohm substituted as source a thermoelectric battery, the principle of which had been discovered by Seebeck in 1821. Using strips of copper and bismuth joined at their two ends, Ohm kept one point of contact in boiling water and the other in ice, and thereby obtained a very stable current in any external circuit he connected across the two points of contact. A magnetic needle was placed over the circuit and suspended from a torsion balance so that the current strength could be gauged by the torsion needed in the balance in order to preserve the pointing direction of the needle.

In one series of experiments, Ohm prepared eight copper conductors of common cross section but different lengths and placed them in turn across the battery, observing that current flow was inversely proportional to wire length. He then considered wires of different material and different diameter and established the general validity of the formula

$$V = IR \qquad R = \rho \frac{l}{A} \tag{8}$$

with R the resistance to current flow, l and A the wire length and cross-sectional area, and ρ a constant whose value depended on the material being used.

Not yet satisfied, Ohm next made the important generalization that the law he had discovered applied to any part of the circuit as well as to the entire length of wire. He compared the flow of electricity to the flow of heat, and drew the parallel that electroscopic force played the same role with respect to current that temperature did with respect to heat conduction. However, neither Ohm nor his contemporaries truly appreciated the relation between the electroscopic force of a battery and the electrostatic potential of Poisson. Several decades were to pass before this relation was widely understood [31] and Ohm was forced to endure a long, bitter period during which the true value of his work was neither recognized nor rewarded.

The law which connects the current flowing in a metallic conductor to the heat evolved was determined by J. P. Joule (1818–1889) in the year 1841 [32]. This was accomplished by coiling wires of different lengths, cross sections, and composition onto thin glass tubes, and then immersing the resulting assemblies in separate beakers containing measured quantities of water. When the same intensity of steady current was passed through the different coils, the water was found to heat up to an equilibrium temperature which differed among the several beakers, but in such a way that the change in temperature was proportional to the resistance of the coil in question. From this Joule concluded

> that when a given quantity of voltaic electricity is passed through a metallic conductor for a given length of time, the quantity of heat evolved by it is always proportional to the resistance which is presents, whatever may be the length, thickness, shape or kind of that metallic conductor.

Joule then reasoned:

> On considering the above law, I thought that the effect produced by the increase of the intensity of the electric current would be as the square of that element, for it is evident that in that case the resistance would be augmented in a double ratio, arising from the increase of the *quantity* of electricity passed in a given time, and also from the increase of the *velocity* of the same. We shall immediately see that this view is actually sustained by experiment.

Finally, we reach the seminal discoveries of Faraday and the role they played in Maxwell's formulation of an electromagnetic theory. Michael Faraday (1791–1867), the son of a struggling blacksmith, is arguably the finest experimentalist in the history of science. His formal education ceased at the age of 13, when he was apprenticed to a bookseller, but an intense thirst for knowledge resulted in a career-long period of self-education. At the age of 20 he became assistant to Sir Humphrey Davy at the Royal Institution and began to establish himself as a premier chemist. He discovered benzene, studied the anomalous behavior of chlorine, undertook an extensive examination of heavy lead oxide glasses, and delved into the alloying of steel to improve its hardness. Faraday also established the fundamental laws of electrolysis, which bear his name.

But it is his contributions to electromagnetics which concern us here. Mention has already been made of his discovery in 1821 of the principle of the electric motor. The prior work of Oersted, Biot and Savart, and Ampère had demonstrated that an electric current could produce magnetic effects. Shouldn't the converse also be true? Faraday attacked this problem many times without success. His laboratory notebook contains an entry dated December 28, 1824, describing an experiment in which a magnet was placed inside a helical coil "but in no case did the magnet seem to affect the current so as to alter its intensity as shewn upon a magnetic needle placed under a distant part of it."

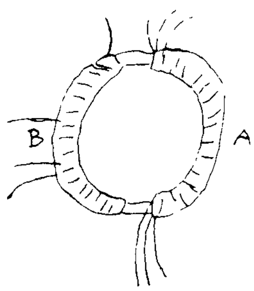
Again, on November 28, 1825, his laboratory notes refer to a battery-connected wire "parallel to which was another similar wire separated from it only by two thicknesses of paper. The ends of the latter wire attached to a galvanometer exhibited no action." Replacing either straight wire by a helix also had no effect.

A third try was recorded on April 22, 1828. Faraday suspended a copper ring by a thread and placed a bar magnet inside the ring but could detect no induced current.

Faraday's efforts were paralleled by those of many other scientists, but no one was having any appreciable measure of success. The difficulty lay in the fact that everyone was looking for the creation of a *steady* current. Perhaps the most significant discovery had been made by Arago in 1824. He suspended a magnetic compass needle over a copper plate and set it into oscillation, noting that the presence of the copper plate enhanced the damping. Upon eliminating air disturbances and rotating the copper plate, Arago was able to make the needle revolve also, and even showed that this dragging effect depended on the conductivity of the rotating plate. Faraday repeated Arago's experiment in 1825 but, despite the suggestiveness of the results, the true explanation of the phenomenon eluded both investigators.

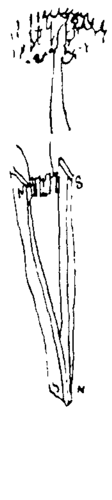
Finally, on August 29, 1831, six years after his first attempt, Faraday discovered the effect he had been seeking. His notes for that day state [33]:

> Have had an iron ring made (soft iron), iron round and 7/8 inches thick and ring 6 inches in external diameter. Wound many coils of copper wire round one half, the coils being separated by twine and calico—there were three lengths of wire each about 24 feet long and they could be connected as one length or used as separate lengths. By trial with a trough each was insulated from the other. Will call this side of the ring A. On the other side but separated by an interval was wound wire in two pieces together amounting to about 60 feet in length, the direction being as with the former coils; this side call B. Charged a battery of 10 pr. plates 4 inches square. Made the coil on B side one coil and connected its extremities by a copper wire passing to a distance and just over a magnetic needle (3 feet from iron ring). Then connected the ends of one of the pieces on A side with battery; immediately a sensible effect on needle. It oscillated and settled at last in original position. On *breaking* connection of A side with Battery again a disturbance of the needle. Made all the wires on A side one coil and sent current from battery through the whole. Effect on needle much stronger than before.



This discovery of transformer action quickly led Faraday to an appreciation of the entire effect. On September 24 he tried a different experiment. Using a remote helix and compass needle as indicator, he wrapped a helical coil around a soft iron cylinder and built up an apparatus which he described as follows:

> The iron cylinder and helix....All the wires made into one helix and these connected with the indicating helix at distance by copper wire: then the iron placed between the poles of bar magnets as... in fig. Every time the magnetic contact at N or S was made or broken there was magnetic motion at the indicating helix, the effect being as in former cases not permanent,



> but a mere momentary push or pull. But if the electric communication (i.e., by the copper wire) was broken then these disjunctions and contacts produced no effect whatever. Hence here distinct conversion of Magnetism into Electricity.

On October 1 Faraday repeated the transformer experiment but with a wooden core, and once again obtained the same effect, though enough weaker that he had to substitute a galvanometer for the indicating helix. He concluded: "Hence there is an inducing effect without the presence of iron."

Finally, on October 17, Faraday performed the most significant experiment of all. He prepared a helical wire in the form of a cylinder and then

> a cylindrical bar magnet $\frac{3}{4}$ inch in diameter and $8\frac{1}{2}$ inches in length had one end just inserted into the end of the helix cylinder—then it was quickly thrust in the whole length and *the galvanometer* needle moved—then pulled out and again the *needle moved but* in the opposite direction. This effect was repeated every time the magnet was put in or out and therefore a wave of Electricity was so produced from *mere approximation of a magnet* and not from its formation *in situ*.

Faraday preferred to think of all electric and magnetic effects in terms of lines of force, having been first attracted to this view by observing the disposition of iron filings in the neighborhood of a permanent magnet. He thus sought to explain this new phenomenon of induced electricity in terms of an interaction with magnetic flux lines. His raw thoughts on this subject are contained in an entry in the laboratory notebook dated August 1, 1851, which contains the passages

> The force of a given magnet is definite and may be considered as represented by its curves....The curves...exist within the magnet as well as without; but within they are in the contrary or return direction....Whatever the condition of the interior of the magnet: it has...the same kind and amount of power as the outside, and so is in full analogy and similitude with an electro helix.
>
> The intensity of the curves of a magnet vary greatly at different distances from the magnet....But the amount of force is definite and the same for every section of all the curves.
>
> Hence it follows that whether the curves are intersected directly or obliquely makes no difference provided they are intersected. The effect depends upon the number of curves intersected. A wire moving obliquely may intersect fewer curves and therefore have a feebler current evolved in it; but if it intersected only the same curves directly across, it would have no larger a current.
>
> So with a given moving wire or with a given wire under which a magnet is moving, the quantity of electricity generated is directly as the amount of curves passed over or through. With the same curves therefore it varies directly with the velocity of the motion.

This explanation of induction as being due to the relative motion between magnetic lines of force and a conductor was refined by Faraday and included in a paper read to the Royal Society later that year [34]. It was given mathematical articulation by Maxwell as the equation

$$e = -\frac{d}{dt}\int_s B_n \, dS \qquad (9)$$

in which e is the emf induced in a contour C and $\int_s B_n \, dS$ is the total magnetic flux enclosed by C. If the contour C is occupied by a conductor, e is the source of the resulting induced electric current. In the above, S is an open surface erected on C as boundary, and B_n is the normal component of flux density, thus representing the number of magnetic lines of force per unit area. This famous equation is known as Faraday's emf law.

After his initial discovery of induction, Faraday continued to experiment with the phenomenon. On October 28, 1831, he invented the first direct-current generator, consisting of a copper plate rotating between magnetic poles, with an external circuit attached between the center and rim of the plate. Through the years Faraday designed and tested a variety of such generators, and his entry for October 11, 1851, describes a machine consisting of a rotating wire rectangle with a *commutator* attached, this being the prototype of the modern electric generator.

Faraday also discovered the phenomenon of self-induction (in 1834), unaware that Joseph Henry (1797–1878) had made an independent discovery of the effect two years earlier.[6]

Faraday also made a fundamental contribution to our understanding of the behavior of dielectric materials. Although Cavendish was the first to observe that the presence of an insulator between the plates of a condenser increased its capacity to store charge for a given voltage, his papers were still unpublished in 1837 when Faraday rediscovered the effect. He measured what today we call the relative dielectric constant for a variety of insulators, both liquid and gaseous, as well as solid. Faraday proposed a model [35] to explain this effect, saying:

> The particles of an insulating dielectric whilst under induction may be compared to a series of small magnetic needles, or more correctly still to a series of small insulated conductors. If the space round a charged globe were filled with a mixture of an insulating dielectric, as oil of turpentine or air, and small globular conductors, as shot, the latter being at a little distance from each other so as to be insulated, then these would in their condition and action exactly resemble what I consider to be the condition and action of the particles of the insulating dielectric itself. If the globe were charged, these little conductors would all be polar; if the globe were discharged, they would all return to their normal state, to be polarized again upon the recharging of the globe.

This insight is all the more remarkable when one remembers the primitive state of atomic theory in 1838. With this model, Faraday was able to deduce that the polarization of the dielectric would be opposite to the influence causing it, thus requiring more primary charge to maintain the same voltage. This provided an explanation for the increase of capacity due to the presence of a dielectric.

In drawing this analogy between dielectric polarization and the behavior of small magnetic needles, Faraday established a link to Poisson's successful theory of magnetization, promulgated 14 years earlier [21]. Equation (5) then became applicable to dielectric materials merely by replacing $\boldsymbol{M}$ by $\boldsymbol{P}$, the latter being the polarization density of the dielectric.

Faraday's interest was also drawn to a study of magnetic materials and he established the distinction between diamagnetic and paramagnetic behavior [36]. Two years later, Wilhelm Weber (1804–1890) offered a detailed explanation [37] of diamagnetism. He assumed the existence of Ampèrian molecular circuits, and invoked Faraday's emf law to argue that currents should be induced in these circuits if a time-varying magnetic field were applied. Since the induction would result in currents whose fields were *opposed* to the stimulus, this would neatly account for diamagnetic behavior.

According to this argument, *all* bodies exhibit diamagnetism. Weber accepted this conclusion, and then assumed further that paramagnetic substances additionally possessed *permanent* molecular currents which were the cause of their paramagnetism. A material whose permanent molecular currents were large would be normally magnetic to such a high degree that the weak diamagnetic effect due to induced currents would be masked completely. Weber was so satisfied with this explanation that he used it as a reason to reject the Coulomb–Poisson hypothesis of polarizable magnetic fluids, saying in the same article:

> Through the discovery of diamagnetism, the hypothesis of electric molecular currents in the interior of bodies is corroborated; the hypothesis of magnetic fluids in the interior of bodies is refuted.

Faraday's researches into the properties of materials followed by a decade or more his discovery of the law of induction. Meanwhile, others repeated and confirmed his various induction experiments. However, his explanation in terms of lines of force fell mainly on deaf ears. The scientists of his day had been reared on theories of action at a distance, theories which had enjoyed wide success in describing a variety of electric and magnetic phenomena, as well as gravitational effects. The eminent Astronomer Royal, Sir George Biddell Airy, declared that he could "hardly imagine any one who knows the agreement between observation and calculation based on action at a distance to hesitate an instant between this simple and

[6] In fairness to Henry, it should be stated that during this period he and Faraday independently discovered many important electromagnetic phenomena, including self- and mutual induction and many of the principles of electric machines. Henry also developed the electromagnetic relay, perfected an electromagnetic telegraph, and showed that voltage could be stepped up or down by properly proportioning the coils in a transformer. Henry's lack of promptness in announcing the results of his experiments has probably been the primary cause of his neglect, but the remoteness of the New World from the Old, in those days of slow communications, was a contributing factor. Faraday's achievements were more promptly disseminated to the European centers of learning, and news of Henry's accomplishments often bore the appearance of mere confirmation of what Faraday had already done. In the stimulation of further scientific inquiry by others, Faraday's influence was inestimably greater.

precise action on the one hand and anything so vague and varying as lines of force on the other."

James Clerk Maxwell (1831–1879), son of a Scottish laird of modest means, educated at Edinburgh and Cambridge, was only 24 when he undertook to overcome this objection and place Faraday's ideas on a firm mathematical basis. In the introduction to his first paper on electricity, he stated that [38]

> the limit of my design is to show how, by a strict application of the ideas and methods of Faraday, the connection of the very different orders of phenomena which he has discovered may be clearly placed before the mathematical mind.

After defining a single line of force as a curve in space whose direction at each point is that of the force on a positive charge, or the force on an elementary north magnetic pole, whichever the case may be, Maxwell continued

> We might in the same way draw other lines of force, till we had filled all space with curves indicating by their direction that of the force at any assigned point.
>
> We should thus obtain a geometrical model of the physical phenomena, which would tell us the *direction* of the force, but we should still require some method of indicating the *intensity* of the force at any point. If we consider these curves not as mere lines, but as fine tubes of variable section carrying an incompressible fluid, then, since the velocity of the fluid is inversely as the section of the tube, we may make the velocity vary according to any given law, by regulating the section of the tube, and in this way we might represent the intensity of the force as well as its direction by the motion of the fluid in these tubes.

Maxwell then pointed out that if the force law involves distance to the inverse square, there would be no interstices between his tubes of force.

> The tubes will then be mere surfaces, directing the motion of a fluid filling up the whole space. It has been usual to commence the investigation of the laws of these forces by at once assuming that the phenomena are due to attractive or repulsive forces acting between certain points. We may, however, obtain a different view of the subject, and one more suited to our difficult inquiries, by adopting for the definition of the forces of which we treat, that they may be represented in magnitude and direction by the uniform motion of an incompressible fluid.

With this conception, Maxwell proceeded to show that all results obtained for static charges or permanent magnets, using action-at-a-distance formulas, were also obtainable in terms of the distribution of tubes of force. Upon pointing out the equivalence of a steady current element and a magnetic dipole, he was also able to extend this conclusion to magnetic phenomena caused by time-independent currents. However, in discussing induced electric currents, Maxwell admitted

> The idea of the electro-tonic state,[7] however, has not yet presented itself to my mind in such a form that its nature and properties may be clearly explained without reference to mere symbols, and therefore I propose in the following investigation to use symbols freely, and to take for granted the ordinary mathematical operations. By a careful study of the laws of elastic solids and of the motions of viscous fluids, I hope to discover a method of forming a mechanical conception of this electro-tonic state adapted to general reasoning.

Maxwell then concluded this first paper with an extensive mathematical development in which the vector potential emerged as being representative of the electrotonic state, its curl giving the magnetic field, and its time derivative yielding the induction effect. He also showed that the curl of the magnetic field at any point was equal to the current density at that point.

This first electrical paper by Maxwell can fairly be described as principally achieving mathematical expression for all known electric and magnetic phenomena in terms of Faraday's physical conceptions. It exhibits Maxwell's characteristic fondness for models, a fondness which had led him to construct a top to illustrate the dynamics of a rigid body rotating about a fixed point, and to construct a model of Saturn's rings (now in the Cavendish Laboratory) to illustrate the motion of the satellites in the rings. This rich physical imagination was now to lead Maxwell to his most important discovery, through an extension of the tube of force model so as to explain the electrotonic state. This extension was accomplished in a second paper, which appeared six years later in the *Philosophical Magazine*, in which he offered the introductory remark [39]

> I propose now to examine magnetic phenomena from a mechanical point of view, and to determine what tensions in, or motions of, a medium are capable of producing the mechanical phenomena observed. If, by the same hypothesis, we can connect the phenomena of magnetic attraction with electromagnetic phenomena and with those of induced currents, we shall have found a theory which, if not true, can only be proved to be erroneous by experiments which will greatly enlarge our knowledge of this part of physics.

It has already been noted that Faraday looked upon electrostatic and magnetic induction as taking place along curved lines of force. Maxwell imagined these lines to be ropes of molecules starting from a charged conductor or magnet, and acting on other nearby bodies. These ropes of molecules were in tension, tending to shorten and at the same time bulge out laterally. Thus the charged conductor or magnet tends to draw bodies to itself, contracting its lines of force like the fibers of a muscle. Maxwell sought to represent this longitudinal tension and transverse pressure in terms of equivalent conditions in a fluid medium.

> Let us now suppose that the phenomena of magnetism depend on the existence of a tension in the direction of the lines of force, combined with a hydrostatic pressure; or in other words, a pressure greater in the equatorial than in the axial direction: the next question is, what mechanical explanation can we give of this inequality of pressures in a fluid or mobile medium? The explanation which most readily occurs to the mind is that the excess of pressure in the equatorial direction arises from

[7]Faraday called the state into which any body was thrown, due to the presence of a magnetic field, the electrotonic state, and explained induction as being due to changes in the electrotonic state.

the centrifugal force of vortices or eddies in the medium having their axes in directions parallel to the lines of force.

We shall suppose at present that all the vortices in any one part of the field are revolving in the same direction about axes nearly parallel, but that in passing from one part of the field to another, the direction of the axes, the velocity of rotation, and the density of the substance of the vortices are subject to change. We shall investigate the resultant mechanical effect upon an element of the medium, and from the mathematical expression of this resultant we shall deduce the physical character of its different component parts.

In order to have adjacent vortices rotating in the same direction, Maxwell next supposed that there exists between them a large number of minute spherical bodies which roll, without sliding, in contact with the surfaces of the vortices. These particles, which Maxwell assumed to constitute electricity, thus play the role of idler wheels. Under this construction, for example, the static magnetic field of a permanent magnet can be envisioned as consisting of vortices which fill the tubes of force, with the rotational velocity of a vortex proportional to the strength of the field and thus varying with tube cross section. With adjacent vortices in the magnetic field rotating at the same speed in the same direction, the particles between them rotate idly but remain in the same position. However, if a change should occur in the magnetic field, this would mean that one of the vortices began rotating faster than the other, and thus the particles between them would change position, indicating an electric current. In this way, Maxwell's model demonstrated the creation of electric currents due to changes in the magnetic field; hydrodynamical considerations of the relations between the rotational velocities of adjacent vortices and the displacement of the idler particles led to a mathematical statement of Faraday's emf law.

It was precisely at this point that the great value of the model became apparent. If a change in vortex motion can cause a displacement of the idler particles, then the converse should be true—a displacement of the idler particles should occasion a change in vortex motion. Cause and effect are interchangeable. A changing magnetic field can create an electric field; a changing electric field should produce a magnetic field. Maxwell was reaching the heart of his greatest contribution when, in part 3 of the paper, he said [40].

According to our theory, the particles which form the partitions between the cells (vortices) constitute the matter of electricity. The motion of these particles constitutes an electric current; the tangential force with which the particles are pressed by the matter of the cells is electromotive force, and the pressure of the particles on each other corresponds to the tension or potential of the electricity.

If we can now explain the condition of a body with respect to the surrounding medium when it is said to be "charged" with electricity, and account for the force acting between electrified bodies, we shall have established a connection between all the principal phenomena of electrical science.

After pointing out that electromotive force (voltage due to magnetic effects) is the same thing as electric tension (voltage due to charge separation), Maxwell distinguished between conductors and insulators, concluding

Here then we have two independent qualities of bodies, one by which they allow of the passage of electricity through them, and the other by which they allow of electrical action being transmitted through them without any electricity being allowed to pass. A conducting body may be compared to a porous membrane which opposes more or less resistance to the passage of a fluid, while a dielectric is like an elastic membrane which may be impervious to the fluid, but transmits the pressure of the fluid on one side to that on the other.

Maxwell next discussed the relation between conduction current and potential in a conductor and then went on to say

Electromotive force acting on a dielectric produces a state of polarization of its parts.... In a dielectric under induction, we may conceive that the electricity in each molecule is so displaced that one side is rendered positively, and the other negatively electrical, but that the electricity remains entirely connected with the molecule, and does not pass from one molecule to another.

The effect of this action on the whole dielectric mass is to produce a general displacement of the electricity in a certain direction. This displacement does not amount to a current, because when it has attained a certain value it remains constant, but it is the commencement of a current, and its variations constitute currents in the positive or negative direction, according as the displacement is increasing or diminishing. The amount of the displacement depends on the nature of the body, and on the electromotive force.

Thus Maxwell introduced for the first time the concept that variations in position of bound charge were equivalent in their effect to a conduction current. By letting motion of the idler particles of his model represent either or both, and finding the variation in vortex velocity due to a particle displacement, he arrived at a generalization of Ampère's circuital law.

The importance of this generalization cannot be overstated. If motion of the idler particles could only represent conduction current, then an electrical disturbance could only propagate through a conductive medium. But with the concept of displacement current, field changes could be transmitted through dielectric media, including air, and even including free space (which Maxwell considered to be an ether).

Maxwell recognized that a finite velocity would be associated with the propagation of any disturbance through his model medium. He described the mechanism of propagation by imagining that a translational motion of one layer of idler particles would initiate a change in angular velocity of the contiguous vortices. These in turn would set the next layer of idler particles into translational motion, and in this manner the disturbance would be transferred through a sequence of layers. Maxwell computed the kinetic and potential energy which were transferred in this fashion, thus obtaining a velocity of transport. By associating kinetic energy and potential energy with the magnetic and electric fields, respectively, he deduced that the velocity of propagation of an electromagnetic disturbance was governed by the electrostatic permittivity and magnetostatic permeability of the supporting medium. Upon using the values for these constants, determined for air by Kohlrausch and Weber, Maxwell deduced that the velocity of an electromagnetic disturbance should be 193088 mi/sec. He then concluded

> the velocity of light in air, as determined by M. Fizeau, is 195,647 miles per second. The velocity of transverse undulations in our hypothetical medium... agrees so exactly with the velocity of light calculated from the optical experiments of M. Fizeau, that we can scarcely avoid the inference that *light consists in the transverse undulations of the same medium* which is the cause of electric and magnetic phenomena.

This discovery may be likened to an earlier occasion when Newton first tested his law of universal gravitation by making calculations on the distance of the moon. It was Newton's misfortune to use an inaccurate value for the diameter of the earth, and this led to such poor agreement that he put the theory aside for nearly two decades. Maxwell was spared a similar disappointment in that both his value and Fizeau's were in error in the same direction.

It should be remembered that at this time no one had ever wittingly generated or detected electromagnetic waves. The concept was completely new, as was the notion of a displacement current. To link light to these hypothetical phenomena was a flash of brilliance seldom equaled in the history of science.

Maxwell next discarded the model which had served so well as a scaffolding with which to erect his theory, and in a third paper, entitled, "A Dynamical Theory of the Electromagnetic Field," presented the theory completely in electrical terms [41]. The properties of the field are described in terms of 20 equations, which include the relation between displacement current and conduction current, and the continuity equation linking charge to current, as well as what are now known conventionally as Maxwell's equations. This paper was so carefully written that it later appears almost intact in his *Treatise*.

These accomplishments, added to his contributions in color vision and molecular theory, have earned Maxwell the place as the greatest theoretical physicist of the 19th century. But what is not generally appreciated is that he was also an extremely competent experimentalist. Thus it is surprising that he never attempted to validate a key prediction of his theory, the existence of electromagnetic waves. That would not be accomplished until eight years after Maxwell's death, with the brilliant experiments of Heinrich Hertz.

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